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Abstract

In this work several location problems with the common optimization flavor are studied. They naturally appear as generalizations of the well-known clustering problems. Many of these have been researched extensively in the last few decades, e.g. the uncapacitated facility location problem or the uncapacitated k-Center problem, and more recently the uncapacitated k-Median problem. Tasks of this nature arise naturally in many contexts, for example in locating new McDonald's restaurants or renting halls for music concerts.

The focus of this thesis is the design and analysis of approximation algorithms for the NP-hard[1] location problems. Especially, we consider location problems in metric spaces, which play an important role in an field of the approximation algorithms. The main technique is randomized rounding of fractional solutions of linear programs. In some cases we know only LP formulation of non-polynomial size, which is strong enough for the applications. It is not obvious how to get or even verify if a particular fractional solution is feasible for such linear program. To deal with some special example of this task we present a technique called "round-or-separate" (see [1, 2, 4, 3]), which exploits the power of the Ellipsoid Method. But as a consequence this method adds some specific requirements on the algorithm.

Typically in the facility location problems we are given a set C of clients and a set F of facilities. The goal of this study is to open a subset of facilities and connect all clients with facilities in a way which optimizes a certain specific objective function. The clustering problems that are studied in this thesis are the following:

(a) the Knapsack Median where we are given a positive opening cost of each facility and a total budget. We should open a subset of the facilities, such that their total cost is not bigger than the given budget;

(b) the Capacitated k-Median where each facility has a capacity, which is the maximum number of clients it can serve in any feasible solution. An additional difficulty is the upper bound k on the total number of the open facilities;

(c) the k-Level Uncapacitated Facility Location where each facility has some positive opening cost and a level. Each client should be at the beginning of a path, that goes though the open facilities on all consecutive levels and ends in a facility on the highest k-th level.

In the two first of the described variants of the location problems the quality of the solution is measured by the sum of distances from each

[1] Researchers are willing to believe that there do not exists polynomial time exact algorithms for NP-hard problems

client to the facility which serves it. In the last one the quality of the solution is measured in slightly more complicated way. The cost of the solution is the sum of the assignment cost and the facility opening cost.

We explore a number of different techniques to solve these problems and give algorithms, for all the above problems, with constant approximation ratios. We believe that the techniques described in the thesis are quite general and have much wider applications than presented here. In particular, we are convinced that presented techniques could be extended and applied to some more general versions of the presented problems.

References

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